

Anharmonic behavior in Microwave-driven resistivity oscillations in Hall bars

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We analyzed the magnetoresistivity of a two-dimensional electron system excited by microwave radiation in a regime of high intensities and low frequencies. In such a regime, recent experiments show that different features appear in the magnetoresistivity response which suggest an anharmonic behavior. These features consist mainly in distorted oscillations and new resonance peaks at the subharmonics of the cyclotron frequency. We follow the model of microwave-driven electron orbits motion which become anharmonic when the ratio of microwave intensity to microwave frequency is large enough.

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Microwave-Induced Resistivity Oscillations (MIRO)^{1,2} and Zero Resistance States (ZRS)^{3,4} have been continuously attracting much attention in the recent years. From the experimental standpoint, different features and improvements are being implemented in the corresponding experimental configurations^{5,6,7,8,9,10}. Thus experimentalist have been introducing changes in the orientations, frequencies, polarizations and intensities of the different fields involved. Accordingly the results obtained with these configurations are getting more and more striking and difficult to explain. In some cases totally unexpected outcomes have been obtained¹⁰. All these experimental evidences and results establish real challenges which can be regarded as crucial tests for the theoretical models currently available^{11,12,13,14,15,16,17}. Recently more experimental outcomes have joined the group of striking experimental contributions^{18,19}. In this case MIRO's are specifically studied in a regime of high MW intensities and low MW frequencies (w). The remarkable results show distorted profiles in the diagonal magnetoresistivity (ρ_{xx}) oscillations and new resonance peaks at the subharmonics of the cyclotron frequencies. These effects become more important when the ratio MW power to MW frequency is larger and exceed a certain threshold. It is also noticeable that the main oscillations are shifted to lower magnetic field as the latter ratio increases.

In this letter we report on a theoretical explanation to this unexpected behavior of ρ_{xx} response obtained in regime of large MW power to w ratio. We follow the model of MW driven Larmor orbits¹⁶ under this extreme regime. According to this model, the electronic orbit guiding center performs under MW excitation, an oscillating and harmonic motion with the same frequency as MW. Thus, any change in the experimental set-up affecting the electronic orbit motion, will be reflected in the corresponding ρ_{xx} response. A very direct example is the experimental proposal regarding ρ_{xx} response to bichromatic MW radiation⁹ and the corresponding theoretical explanation²⁰. Following this model, if we increase the MW power and decrease w , the amplitude of the Larmor orbit oscillation becomes larger and larger making eventually the oscillating motion *anharmonic*. Then the electrons in their Larmor orbits become *driven anharmonic oscillators*. In this letter we consider only the case of slightly driven anharmonic behavior. It is well-known that a small amplitude driven anharmonic oscillating system has the following characteristics:

(1) the forced oscillations contains harmonics that are not present in the driving force; (2) subsidiary resonances occur at driving frequencies which are subharmonic of the main resonance frequency; and (3) the main oscillations are moved to lower frequencies²¹. All these features are clearly present in the experimental results¹⁸. A possible microscopic mechanism for the anharmonic oscillating motion involves an interplay between a large oscillation amplitude and the interaction between electrons in their MW-driven oscillating Larmor orbits and the lattice ions.

Following the MW driven Larmor orbits model, we first obtain the exact expression of the electronic wave vector for a two-dimensional electron system (2DES) in a perpendicular and moderate magnetic field B , and MW radiation^{16,22}:

$$\begin{aligned} \Psi(x, y, t) = & \phi_N [(x - X - x_{cl}(t)), (y - y_{cl}(t)), t] \\ & \times \exp \frac{i}{\hbar} \left[m^* \left(\frac{dx_{cl}}{dt} x + \frac{dy_{cl}}{dt} y \right) + \frac{m^* w_c (x_{cl} x - y_{cl} y)}{2} - \int_0^t L dt' \right] \\ & \times \sum_{p=-\infty}^{\infty} J_p(A_N) e^{ipwt} \end{aligned} \quad (1)$$

where ϕ_N are analytical solutions for the Schrödinger equation with a two-dimensional (2D) parabolic confinement, known as Fock-Darwin states. X is the center of the orbit for the electron spiral motion. $x_{cl}(t)$ and $y_{cl}(t)$ are the classical solutions for a driven 2D harmonic oscillator. For MW radiation polarized along the current direction (x-direction) and a harmonic oscillation regime, the expression for x_{cl} is given by^{16,22}:

$$x_{cl}(t) = \frac{eE_o}{m^* \sqrt{(w_c^2 - w^2)^2 + \gamma^4}} \cos wt \quad (2)$$

where γ is a sample dependent damping parameter which affects dramatically the MW-driven electronic orbits movement. Along with this movement there occur interactions between electrons and lattice ions yielding acoustic phonons and producing a damping effect in the electronic motion¹⁶. E_o is the amplitude of the MW field. L is the classical lagrangian, and J_p are Bessel functions^{16,22,23}.

In the present regime, x_{cl} and y_{cl} are the solutions for the dynamics of a *2D driven classical anharmonic oscillator*. Since we do not know the exact nature of the anharmonic term in the corresponding potential is impossible to solve analytically the classical equation of motion²⁴. Nevertheless we can take an alternative approach if we consider that

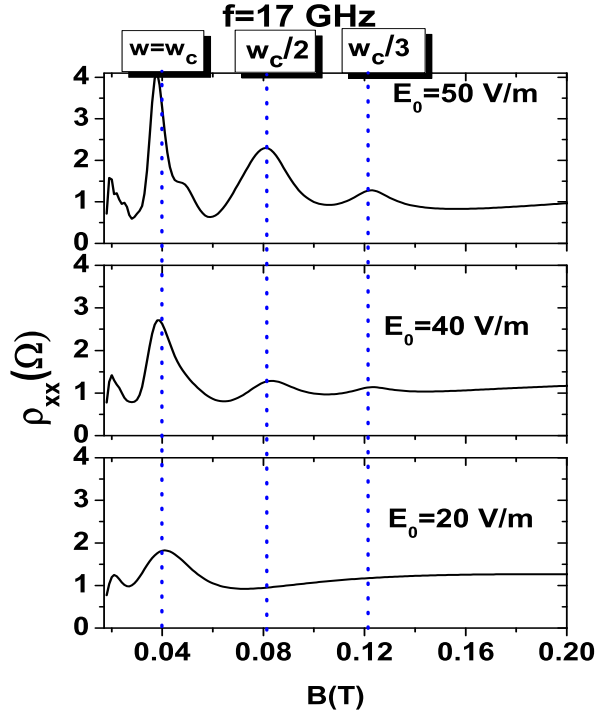


FIG. 1: Calculated magnetoresistivity ρ_{xx} as a function of B for different MW intensities. MW frequency is $17GHz$. Vertical dashed lines correspond to the position of the cyclotron resonance ($w = w_c$) and its subharmonics. It can be observed very clearly the increasing anharmonicity of ρ_{xx} as the MW power is getting larger. $T=1K$.

the oscillating orbits are only slightly anharmonic. Then, although not harmonic, the system can be consider still periodic. As for any periodic function, we can try to express x_{cl} through a Fourier series and propose a solution like:

$$x_{cl}(t) = \frac{A_0}{2} + \sum_{n=1}^{\infty} [A_n \cos(nwt) + B_n \sin(nwt)] \quad (3)$$

where A_0 , A_n and B_n are the corresponding Fourier coefficients.

Now we introduce the scattering suffered by the electrons due to charged impurities randomly distributed in the sample. To proceed, following the model described in reference [16], firstly we calculate the electron-charged impurity scattering rate $1/\tau$ (being τ the scattering time). Secondly we find the average effective distance advanced by the

electron in every scattering jump, that in the case of anharmonicity is given by:

$$\Delta X^{MW} = \Delta X^0 + A_0/2 + \sum_n [A_n \cos(nw\tau) + B_n \sin(nw\tau)] \quad (4)$$

where ΔX^0 is the effective distance advanced when there is no MW field present¹⁶. The longitudinal conductivity σ_{xx} can be calculated: $\sigma_{xx} \propto \int dE \frac{\Delta X^{MW}}{\tau} (f_i - f_f)$, being f_i and f_f the corresponding distribution functions for the initial and final states respectively and E energy. To obtain ρ_{xx} we use the relation $\rho_{xx} = \frac{\sigma_{xx}}{\sigma_{xx}^2 + \sigma_{xy}^2} \simeq \frac{\sigma_{xx}}{\sigma_{xy}^2}$, where $\sigma_{xy} \simeq \frac{n_i e}{B}$ and $\sigma_{xx} \ll \sigma_{xy}$. Finally we can express ρ_{xx} as being proportional to a sum of Fourier terms which reads:

$$\rho_{xx} \propto \sum_n [A_n \cos(nw\tau) + B_n \sin(nw\tau)] \quad (5)$$

In order to obtain the Fourier terms in ρ_{xx} , we have carried out a Fourier synthesis process. This process consists in constructing the ρ_{xx} form by adding together a fundamental frequency (which corresponds to the harmonic case) and overtones of different amplitudes. Since at this stage it is impossible to obtain analytical expressions for the Fourier coefficients, we have introduced phenomenologically the following ones:

$$A_n = \alpha_n \frac{eE_0}{m^* \sqrt{(w_c^2 - (nw)^2)^2 + \gamma^4}} = \alpha_n C_n \text{ and } B_n = \beta_n \frac{eE_0}{m^* \sqrt{(w_c^2 - w^2)^2 + \gamma^4}} = \beta_n C_1 \text{ where } \alpha_n \text{ and } \beta_n \text{ are anharmonicity terms. Their values are getting larger as the anharmonicity increases.}$$

In Fig.1, we show the calculated ρ_{xx} , as a function of B for different MW intensities and $w = 17GHz$. Vertical dashed lines are positioned at the cyclotron resonance ($w = w_c$) and its subharmonics. For the three curves presented different developed expressions of ρ_{xx} have been used. Each one shows the increasing anharmonicity for increasing E_0/w ratio. As we said above, they have been obtained through a Fourier synthesis process where the anharmonicity coefficients have been introduced phenomenologically, keeping the number of Fourier terms as small as possible. Thus for the top panel of Fig. 1:

$$\rho_{xx} \propto C_1 \cos w\tau + 0.8C_2 \cos 2w\tau + 0.2C_3 \cos 3w\tau + C_1 [0.4 \sin 2w\tau + 0.2 \sin 3w\tau + 0.2 \sin 4w\tau] \quad (6)$$

For the middle panel:

$$\begin{aligned} \rho_{xx} \propto C_1 \cos w\tau + 0.4C_2 \cos 2w\tau + 0.1C_3 \cos 3w\tau + \\ C_1[0.2 \sin 2w\tau + 0.1 \sin 3w\tau] \end{aligned} \quad (7)$$

And finally for the bottom panel with the lowest E_0/w ratio we recover the harmonic response¹⁶:

$$\rho_{xx} \propto C_1 \cos w\tau \quad (8)$$

These figures illustrate how the ρ_{xx} profile presents increasing anharmonicity as the MW power is also increased. Thus, it can be observed clearly the anharmonicity features: distorted profile in the ρ_{xx} oscillations, new resonance peaks at the subharmonics of the cyclotron frequencies and finally it is also remarkable that the main oscillations are shifted to lower magnetic fields. All this features corresponds unambiguously to a slightly anharmonic behavior, i.e., the system amplitude is not very large yet. However when a non-linear system is driven with very large amplitude, new vibrational phenomena appear, like vibrations in which the motion only repeat itself after two or more driver periods leading the systems finally into a chaotic regime²⁵. This latter case is not considered in this letter.

In Fig. 2 we present calculated ρ_{xx} versus w_c/w for three different frequencies and MW intensities. We can observe, as in the experiment¹⁸, how the anharmonicity effects increase with decreasing frequency although the MW power also decreases. This suggests that in the evolution to anharmonicity, w plays a more important role than MW power. Thus, according to a driven oscillating system, we propose that the ratio which governs that evolution be proportional to the corresponding driven amplitude:

$$\frac{eE_0}{m^* \sqrt{(w_c^2 - w^2)^2 + \gamma^4}} \propto \frac{E_0}{w^2} \quad (9)$$

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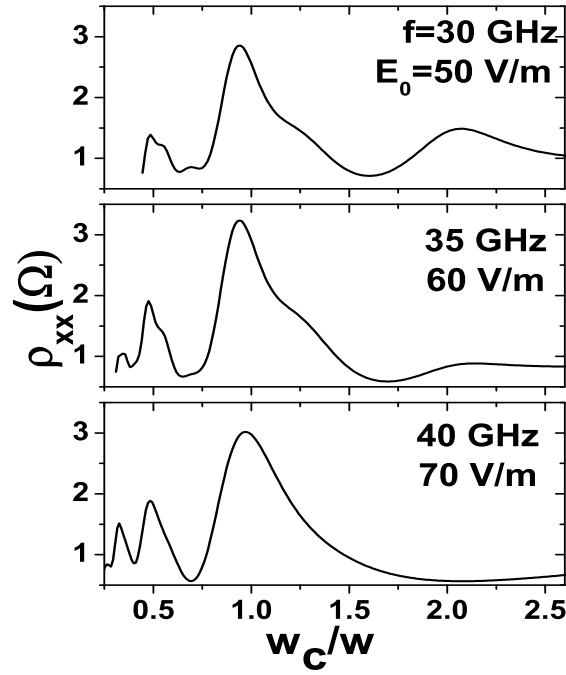


FIG. 2: Calculated ρ_{xx} versus w_c/w for three different frequencies and MW intensities. We can observe how the anharmonicity effects increase with decreasing frequencies. $T=1K$.

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